CHEM-E7225 Advanced Process Control

Seminar – Model Predictive Control of Water Resource Recovery Facilities

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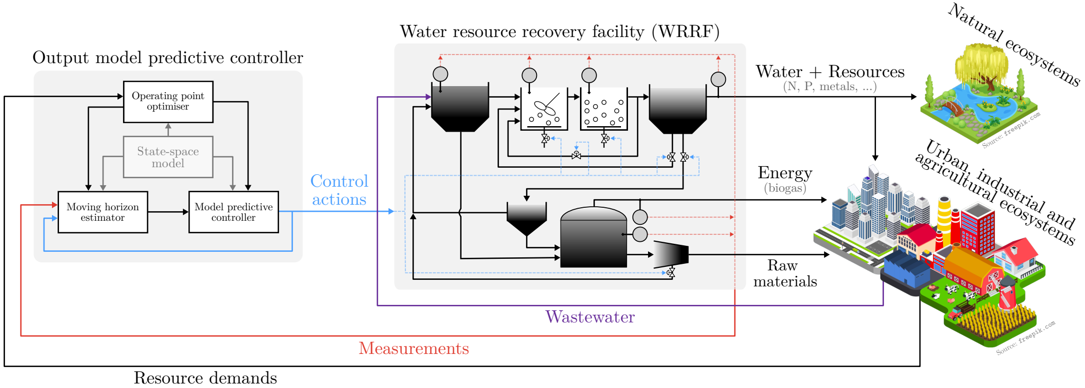
Intro, operating WWTPs as self-sufficient WRRFs

Context: Wastewater as a sustainable source of water, energy, and raw materials.

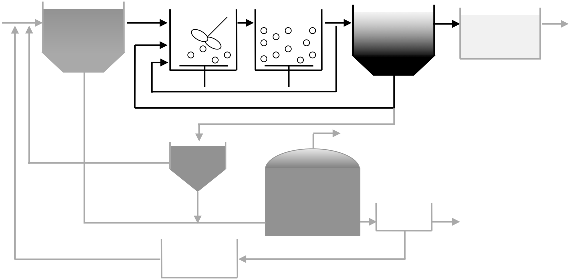
Dynamic demands require flexible operation policies that still ensures a balanced water-energy nexus.

Model-based controllers that adjust the effluent to external demands while recovering energy from sludge.



Intro, activated sludge processes (ASP)

- We focus on the Activated Sludge Processes (ASP) on biological WWTPs



- **Denitrification-nitrification:**
  - ⇝ Bacteria reduce nitrate and ammonia nitrogen into nitrogen gas

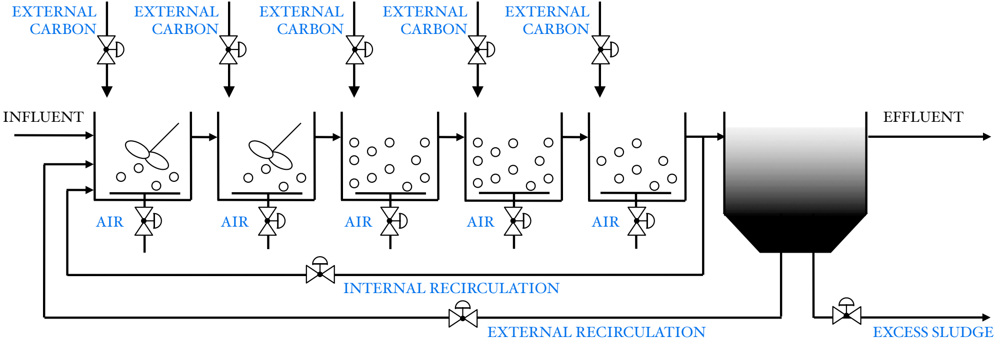
- **Process Manipulation:**
  1. Insufflate air to boost aerobic growth
  2. Extra carbon to boost aerobic/anoxic growth
  3. Recycle wastewater within the plant

Activated sludge process

$$\texttt{Ammonium} + \texttt{Oxygen} \xrightarrow{\text{Autotrophic bacteria}} \texttt{Nitrate} + \texttt{Byproducts}$$

$$\texttt{Nitrate} + \texttt{Carbon} \xrightarrow{\text{Heterotrophic bacteria}} \texttt{Nitrogen gas} + \texttt{Byproducts}$$

Intro, benchmark simulation models



Benchmark Simulation Model no. 1 (BSM1)[1]

⇝ 5 sequential bio-reactors (Activated Sludge Model no. 1)

⇝ 1 non-reactive settler (10-layers double-exponential settling model)

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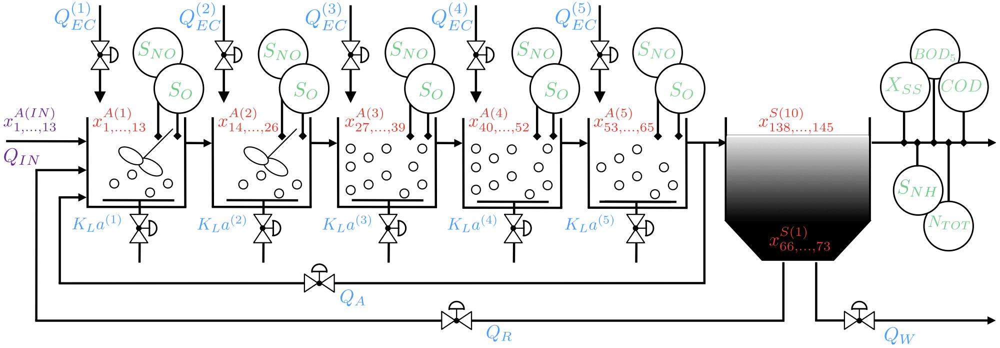
[1] Gernaey, K., Jeppsson, U., Vanrolleghem, P., Copp, J., 2014. Benchmarking of Control Strategies for Wastewater Treatment Plants. IWA.

Intro, benchmark simulation models (cont.)



An “expanded model” when compared to common representations

$\rightsquigarrow$ $N_x = 5 \times 13 + 10 \times 8$ $= 145$ state variables

$\rightsquigarrow$ $N_u = 3 + 5 \times 2$ $= 13$ controls

$\rightsquigarrow$ $N_w = 1 + 13$ $= 14$ disturbances

$\rightsquigarrow$ $N_y = 5 \times 2 + 5$ $= 15$ sensors

$$
\begin{aligned}
\dot{S}_I^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[S_I^{A(r-1)} - S_I^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} S_I^{A(r)} \\
\dot{S}_S^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[S_S^{A(r-1)} - S_S^{A(r)}\Big] + \frac{Q_{EC}^{(r)}}{V^{A(r)}}\Big[S_S^{EC} - S_S^{A(r)}\Big] \\
&\quad - \frac{\mu_H}{Y_H}\frac{S_S^{A(r)}}{K_S + S_S^{A(r)}}\Big[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_g \frac{K_{OH}}{K_{OH} + S_O^{A(r)}}\frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\Big] X_{BH}^{A(r)} \\
&\quad + k_h \frac{X_S^{A(r)}}{K_X X_{BH}^{A(r)} + X_S^{A(r)}}\Big[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_h \frac{K_{OH}}{K_{OH} + S_O^{A(r)}}\frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\Big] X_{BH}^{A(r)} \\
\dot{X}_I^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[X_I^{A(r-1)} - X_I^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} X_I^{A(r)} \\
\dot{X}_S^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[X_S^{A(r-1)} - X_S^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} X_S^{A(r)} \\
&\quad - k_h \frac{X_S^{A(r)}}{K_X X_{BH}^{A(r)} + X_S^{A(r)}}\Big[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_h \frac{K_{OH}}{K_{OH} + S_O^{A(r)}}\frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\Big] X_{BH}^{A(r)} \\
&\quad + \Big[1 - f_p\Big] b_H X_{BH}^{A(r)} + \Big[1 - f_p\Big] b_A X_{BA}^{A(r)} \\
\dot{X}_{BH}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[X_{BH}^{A(r-1)} - X_{BH}^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} X_{BH}^{A(r)} \\
&\quad + \mu_H \frac{S_S^{A(r)}}{K_S + S_S^{A(r)}}\Big[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_g \frac{K_{OH}}{K_{OH} + S_O^{A(r)}}\frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\Big] X_{BH}^{A(r)} - b_H X_{BH}^{A(r)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{X}_{BA}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\left[X_{BA}^{A(r-1)} - X_{BA}^{A(r)}\right] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} X_{BA}^{A(r)} \\
&\quad + \mu_A \frac{S_{NH}^{A(r)}}{K_{NH} + S_{NH}^{A(r)}} \frac{S_O^{A(r)}}{K_{OA} + S_O^{A(r)}} X_{BA}^{A(r)} - b_A X_{BA}^{A(r)} \\
\dot{X}_{P}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\left[X_{P}^{A(r-1)} - X_{P}^{A(r)}\right] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} X_{P}^{A(r)} + f_P\left[b_H X_{BH}^{A(r)} + b_A X_{BA}^{A(r)}\right] \\
\dot{S}_{O}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\left[S_{O}^{A(r-1)} - S_{O}^{A(r)}\right] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} S_{O}^{A(r)} + K_L a^{(r)}\left[S_O^{sat} - S_O^{A(r)}\right] \\
&\quad - \frac{1 - Y_H}{Y_H}\mu_H \frac{S_S^{A(r)}}{K_S + S_S^{A(r)}} \frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} X_{BH}^{A(r)} - \frac{4.57 - Y_A}{Y_A}\mu_A \frac{S_{NH}^{A(r)}}{K_{NH} + S_{NH}^{A(r)}} \frac{S_O^{A(r)}}{K_{OA} + S_O^{A(r)}} X_{BA}^{A(r)} \\
\dot{S}_{NO}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\left[S_{NO}^{A(r-1)} - S_{NO}^{A(r)}\right] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} S_{NO}^{A(r)} \\
&\quad - \frac{1 - Y_H}{2.86 Y_H}\mu_H \frac{S_S^{A(r)}}{K_S + S_S^{A(r)}} \frac{K_{OH}}{K_{OH} + S_O^{A(r)}} \frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}} \eta_g X_{BH}^{A(r)} + \frac{\mu_A}{Y_A} \frac{S_{NH}^{A(r)}}{K_{NH} + S_{NH}^{A(r)}} \frac{S_O}{K_{OA} + S_O^{A(r)}} X_{BA}^{A(r)} \\
\dot{S}_{NH}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\left[S_{NH}^{A(r-1)} - S_{NH}^{A(r)}\right] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} S_{NH}^{A(r)} \\
&\quad - \mu_H \frac{S_S}{K_S + S_S^{A(r)}}\left[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_g \frac{K_{OH}}{K_{OH} + S_O^{A(r)}} \frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\right] i_{XB} X_{BH}^{A(r)} \\
&\quad - \mu_A\left(i_{XB} + \frac{1}{Y_A}\right) \frac{S_{NH}^{A(r)}}{K_{NH} + S_{NH}^{A(r)}} \frac{S_O^{A(r)}}{K_{OA} + S_O^{A(r)}} X_{BA}^{A(r)} + k_a S_{ND}^{A(r)} X_{BH}^{A(r)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{S}_{ND}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[S_{ND}^{A(r-1)} - S_{ND}^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} S_{ND}^{A(r)} \\
&- k_a S_{ND}^{A(r)} X_{BH}^{A(r)} + k_h \frac{X_{ND}^{A(r)} X_{BH}^{A(r)}}{K_X X_{BH}^{A(r)} + X_S^{A(r)}}\Big[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_h \frac{K_{OH}}{K_{OH} + S_O^{A(r)}} \frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\Big] \\
\dot{X}_{ND}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[X_{ND}^{A(r-1)} - X_{ND}^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} X_{ND}^{A(r)} \\
&- k_h \frac{X_{ND}^{A(r)}}{K_X X_{BH}^{A(r)} + X_S^{A(r)}}\Big[\frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} + \eta_h \frac{K_{OH}}{K_{OH} + S_O^{A(r)}} \frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}}\Big] X_{BH}^{A(r)} \\
&+ b_H \Big(i_{XB} - f_P i_{XP}\Big) X_{BH}^{A(r)} + b_A \Big(i_{XB} - f_P i_{XP}\Big) X_{BA}^{A(r)} \\
\dot{S}_{ALK}^{A(r)} &= \frac{Q^{(r)}}{V^{A(r)}}\Big[S_{ALK}^{A(r-1)} - S_{ALK}^{A(r)}\Big] - \frac{Q_{EC}^{(r)}}{V^{A(r)}} S_{ALK}^{A(r)} \\
&- \frac{i_{XB}}{14} \mu_H \frac{S_S^{A(r)}}{K_S + S_S^{A(r)}} \frac{S_O^{A(r)}}{K_{OH} + S_O^{A(r)}} X_{BH}^{A(r)} + \frac{1}{14} k_a S_{ND}^{A(r)} X_{BH}^{A(r)} \\
&+ \Big(\frac{1 - Y_H}{14 \times 2.86 Y_H} - \frac{i_{XB}}{14}\Big) \mu_H \frac{S_S^{A(r)}}{K_S + S_S^{A(r)}} \frac{K_{OH}}{K_{OH} + S_O^{A(r)}} \frac{S_{NO}^{A(r)}}{K_{NO} + S_{NO}^{A(r)}} \eta_g X_{BH}^{A(r)} \\
&- \Big(\frac{i_{XB}}{14} + \frac{1}{7 Y_A}\Big) \mu_A \frac{S_{NH}^{A(r)}}{K_{NH} + S_{NH}^{A(r)}} \frac{S_O^{A(r)}}{K_{OA} + S_O^{A(r)}} X_{BA}^{A(r)}
\end{aligned}
$$

Dynamics of particulate matter[1]

$$\dot{X}_{SS}^{S(l)} = \begin{cases} \frac{Q_e}{V^{S(l)}}\left[X_{SS}^{S(l-1)} - X_{SS}^{S(l)}\right] - \frac{1}{h^{S(l)}} J_{cla}\left(X_{SS}^{S(l)}, X_{SS}^{S(l-1)}\right) & (l = 10) \\ \frac{Q_e}{V^{S(l)}}\left[X_{SS}^{S(l-1)} - X_{SS}^{S(l)}\right] + \frac{1}{h^{S(l)}}\left[J_{cla}\left(X_{SS}^{S(l+1)}, X_{SS}^{S(l)}\right) - J_{cla}\left(X_{SS}^{S(l)}, X_{SS}^{S(l-1)}\right)\right] & (l = 7, \cdots, 9) \\ \frac{Q_f}{V^{S(l)}}\left[X_f - X_{SS}^{S(l)}\right] + \frac{1}{h^{S(l)}}\left[J_{cla}\left(X_{SS}^{S(l+1)}, X_{SS}^{S(l)}\right) - J_{st}\left(X_{SS}^{S(l)}, X_{SS}^{S(l-1)}\right)\right] & (l = 6) \\ \frac{Q_u}{V^{S(l)}}\left[X_{SS}^{S(l+1)} - X_{SS}^{S(l)}\right] + \frac{1}{h^{S(l)}}\left[J_{st}\left(X_{SS}^{S(l+1)}, X_{SS}^{S(l)}\right) - J_{st}\left(X_{SS}^{S(l)}, X_{SS}^{S(l-1)}\right)\right] & (l = 2, \cdots, 5) \\ \frac{Q_u}{V^{S(l)}}\left[X_{SS}^{S(l+1)} - X_{SS}^{S(l)}\right] + \frac{1}{h^{S(l)}} J_{st}\left(X_{SS}^{S(l+1)}, X_{SS}^{S(l)}\right) & (l = 1) \end{cases}$$

Dynamics of soluble matter

$$\dot{S}_{(\cdot)}^{S(l)} = \begin{cases} \frac{Q_e}{V^{S(l)}}\left[S_{(\cdot)}^{S(l-1)} - S_{(\cdot)}^{S(l)}\right] & (l = 7, \cdots, 10) \\ \frac{Q_f}{V^{S(l)}}\left[S_{(\cdot)}^{A(5)} - S_{(\cdot)}^{S(l)}\right] & (l = 6) \\ \frac{Q_u}{V^{S(l)}}\left[S_{(\cdot)}^{S(l+1)} - S_{(\cdot)}^{S(l)}\right] & (l = 1, \cdots, 5) \end{cases}$$

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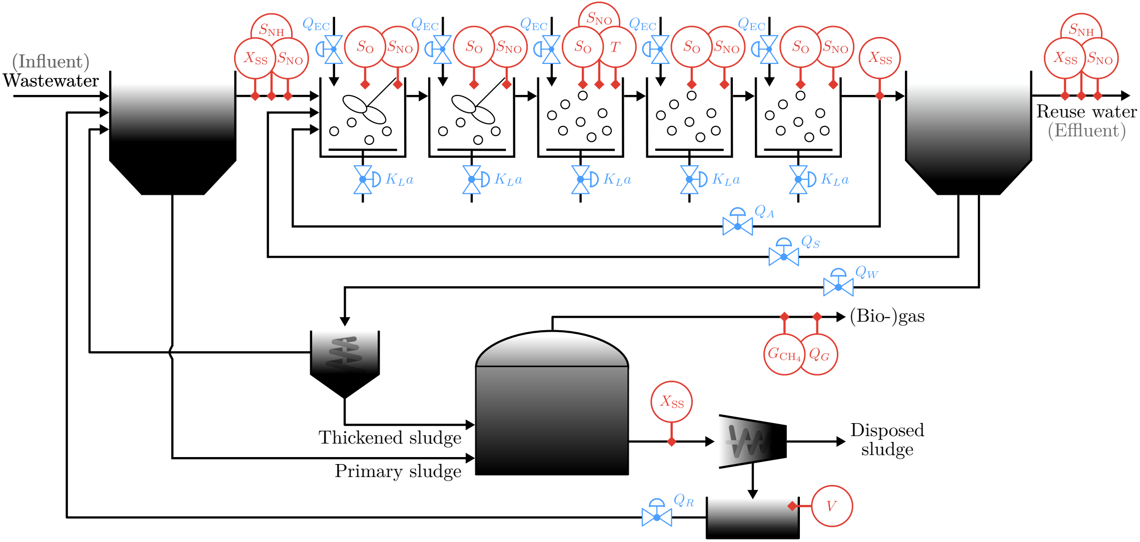
[1]Settling and clarification functions:

$$J_{st}\left(X_{SS}^{S(l)}, X_{SS}^{S(l-1)}\right) = \mathtt{min}\left[v_s\left(X_{SS}^{S(l-1)}\right) X_{SS}^{S(l-1)}, v_s\left(X_{SS}^{S(l)}\right) X_{SS}^{S(l)}\right]$$

$$J_{cla}\left(X_{SS}^{S(l)}, X_{SS}^{S(l-1)}\right) = \begin{cases} \mathtt{min}\left[v_s\left(X_{SS}^{S(l-1)}\right) X_{SS}^{S(l-1)}, v_s\left(X_{SS}^{S(l)}\right) X_{SS}^{S(l)}\right] & \text{if } X_{SS}^{S(l-1)} > X_t \\ v_s\left(X_{SS}^{S(l)}\right) X_{SS}^{S(l)} & \text{otherwise} \end{cases}$$

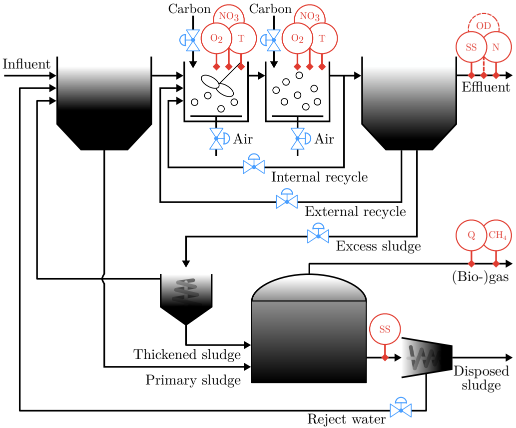
Intro, benchmark simulation models (cont.)



Experimental study

Experimental study, setup

We consider the task of operating a **biological wastewater treatment plant** (WWTP, secondary treatment) as a **water resource recovery facility** (WRRF)



- PRIMARY OBJECTIVE:
  on demand, produce effluent water of specific quality

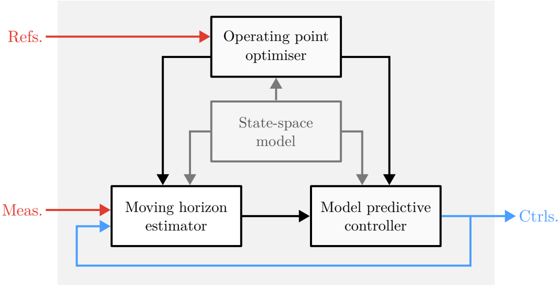
Water class	Biochemical profile		
	TSS	BOD	TN
A	≤ 30 g/m^3	≤ 10 g/m^3	≤ 15 g/m^3
B	≤ 30 g/m^3	≤ 15 g/m^3	≤ 30 g/m^3
C	≤ 30 g/m^3	≤ 20 g/m^3	≤ 45 g/m^3

- SECONDARY OBJECTIVE:
  produce biogas to ensure nonpositive energy cost index,

$$\mathrm{ECI} = \mathrm{AE} + \mathrm{PE} + \mathrm{ME} - \eta_E \mathrm{MP} + \max(0, \mathrm{HE} - \eta_H \mathrm{MP})$$

Experimental study, general control architecture and implementation

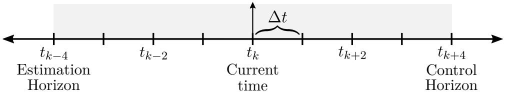
Output model predictive controller



We design a **model-based** output-feedback controller

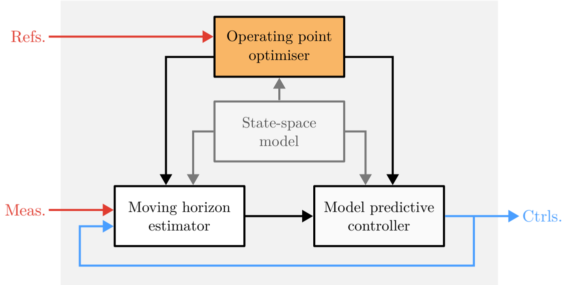
$$\Sigma := \begin{cases} \frac{d}{dt}x(t) = f(x(t), w(t), u(t)) & \text{(dynamics)} \\ y(t) = g(x(t), w(t), u(t)) & \text{(measurements)} \\ z(t) = h(x(t), w(t), u(t)) & \text{(performance)} \end{cases}$$

which autonomously operates the plant in cycles



Experimental study, general control architecture and implementation

Output model predictive controller



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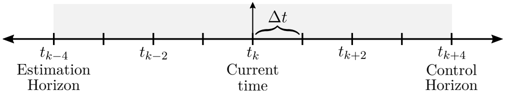


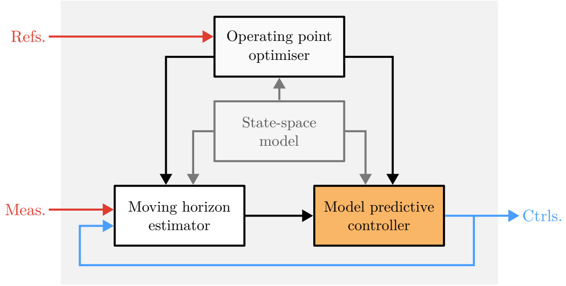
Operating point optimizer — OPO($\cdot$)

$$\begin{aligned} \underset{x_k,\, u_k}{\text{minimize}} \quad & \left\| \begin{bmatrix} W_{z|\text{ref}} & \\ & W_{u|\text{ref}} \end{bmatrix} \begin{bmatrix} z_k - z_k^{\text{ref}} \\ u_k - \bar{u}_k^{\text{ref}} \end{bmatrix} \right\|_2^2 \\ \text{subject to} \quad & 0 = f(x_k, \bar{w}_k^{\text{ref}}, u_k) \\ & z_k \in \mathcal{Z}_{\text{ref}}, \quad x_k \in \mathcal{X}_{\text{ref}}, \quad u_k \in \mathcal{U}_{\text{ref}} \end{aligned}$$

$$\Sigma^{\delta|k} := \begin{cases} zx^{\delta}[n] = A_k x^{\delta}[n] + B_{w|k} w^{\delta}[n] + B_{u|k} u^{\delta}[n] \\ y^{\delta}[n] = C_k x^{\delta}[n] + D_{w|k} w^{\delta}[n] \end{cases}$$

Experimental study, general control architecture and implementation

Output model predictive controller



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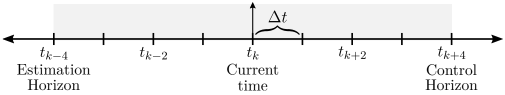
$$\Sigma := \begin{cases} \frac{d}{dt}x(t) = f(x(t), w(t), u(t)) & \text{(dynamics)} \\ y(t) = g(x(t), w(t), u(t)) & \text{(measurements)} \\ z(t) = h(x(t), w(t), u(t)) & \text{(performance)} \end{cases}$$

which autonomously operates the plant in cycles



Model predictive controller — MPC($\cdot$)

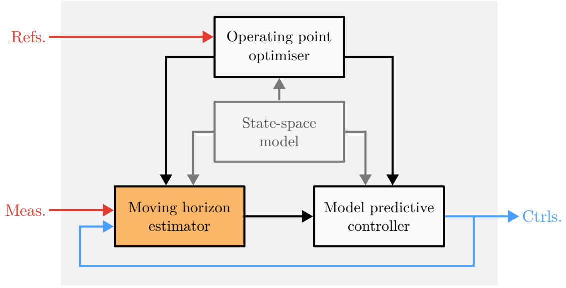
$$\underset{u^\delta[\cdot],\ x^\delta[\cdot]}{\text{minimize}} \quad \sum_{n=0}^{N_c-1} \left\| \begin{bmatrix} W_{x|n} & \\ & W_{u|n} \end{bmatrix} \begin{bmatrix} x^\delta[n] \\ u^\delta[n] \end{bmatrix} \right\|_2^2 + \left\| W_{x|N_c} x^\delta[N_c] \right\|_2^2$$

$$\text{subject to} \quad \Sigma^{\delta|k} \text{ with } w^\delta[n] = \hat{w}^\delta_{N_e-1} \text{ and } x^\delta[0] = \hat{x}^\delta_{N_e}$$

$$x^\delta_{\text{lb}} \preceq x^\delta[n] \preceq x^\delta_{\text{ub}}, \quad u^\delta_{\text{lb}} \preceq u^\delta[n] \preceq u^\delta_{\text{ub}}$$

Experimental study, general control architecture and implementation

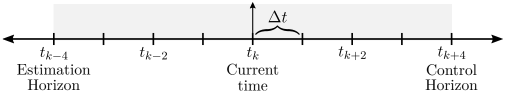
Output model predictive controller



We design a **model-based** output-feedback controller

$$\Sigma := \begin{cases} \frac{d}{dt}x(t) = f(x(t), w(t), u(t)) & \text{(dynamics)} \\ y(t) = g(x(t), w(t), u(t)) & \text{(measurements)} \\ z(t) = h(x(t), w(t), u(t)) & \text{(performance)} \end{cases}$$

which autonomously operates the plant in cycles



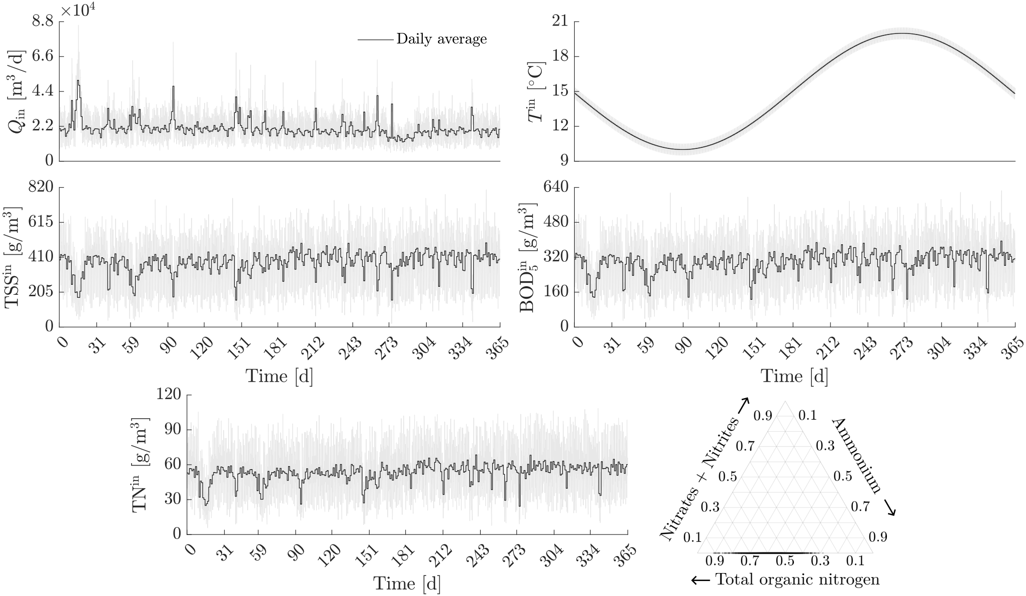
Moving horizon estimator — MHE($\cdot$)

$$\underset{w^\delta[\cdot],\ x^\delta[\cdot]}{\text{minimize}} \quad \sum_{n=0}^{N_e-1} \left\| \begin{bmatrix} W_{y|n} & \\ & W_{w|n} \end{bmatrix} \begin{bmatrix} y^\delta[n] - y_n^{\text{data}|\delta} \\ w^\delta[n] - w_n^{\text{data}|\delta} \end{bmatrix} \right\|_2^2 + \left\| W_{y|N_e} \left(y^\delta[N_e] - y_{N_e}^{\text{data}|\delta} \right) \right\|_2^2$$

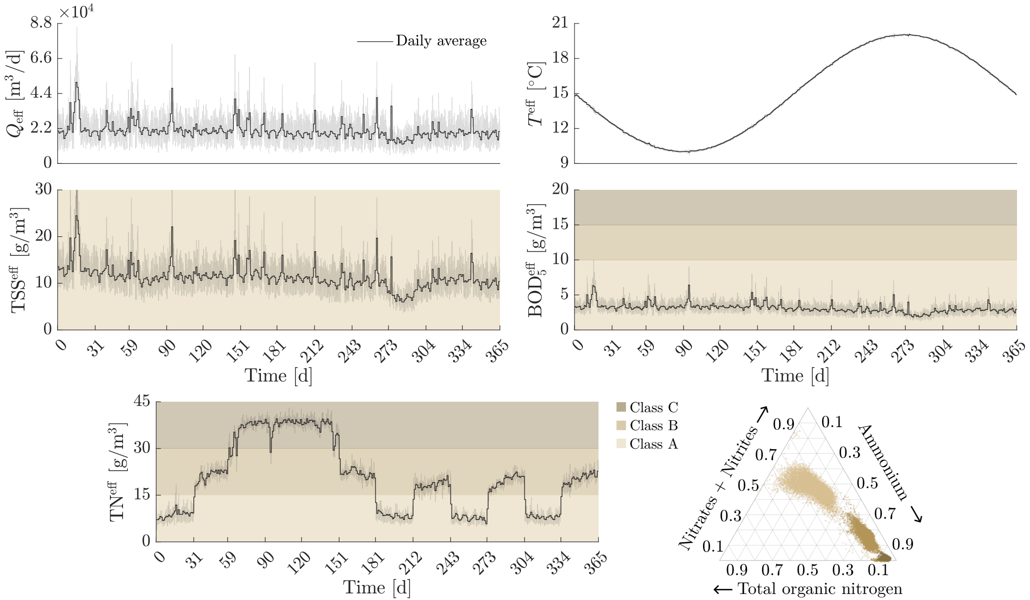
$$\text{subject to} \quad \Sigma^{\delta|k} \text{ with } u^\delta[n] = u_n^{\text{data}|\delta}$$

$$x_{\text{lb}}^\delta \preceq x^\delta[n] \preceq x_{\text{ub}}^\delta, \quad w_{\text{lb}}^\delta \preceq w^\delta[n] \preceq w_{\text{ub}}^\delta.$$

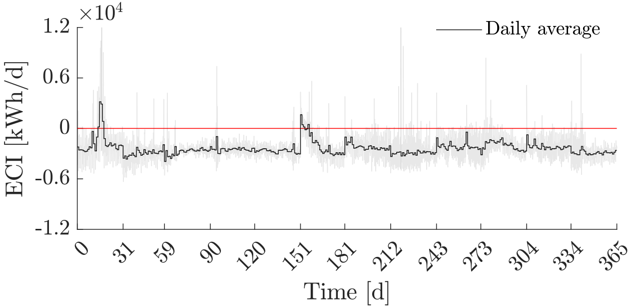
Experimental study, results



Experimental study, results



Experimental study, results



Summary
Total production: 83 GWh
Daily production: 2385 ± 1312 kWh

The controller transitions the biological WWTP to a energetically self-sufficient WRRF

Thank you!

Questions?